# Unitarity Constraints for the Mass of the Higgs in the $SU(2)_L \otimes U(1) \otimes U(1)'$ Gauge Model

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**Abstract**
The critical values for the mass of the Higgs bosons, at which the theory becomes strongly interacting, are calculated using the equivalence theorem, at high energies this allows us to replace the longitudinally polarized gauge bosons in the $S$ matrix for the corresponding Goldstone bosons. An appropriate ansatz for defining the would-be Goldstone bosons in the case of an additional neutral current, beyond the minimal standard model, is also presented.

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The minimal standard model (MSM) gives an adequate description of the available experimental data on the electroweak interaction, but, theorists believe, there is physics beyond the MSM and hope that the advent of new accelerators will show some experimental evidence of this. This new physics should bring the need for new models like the supersymmetric models, technicolour or simple extensions to the gauge group. One can think in many different extensions to the gauge group, some of the most relevant are: the modification of the Higgs sector (including two $SU(2)_L$ Higgs doublets, one additional Higgs singlet, one additional $SU(2)_L$ triplet or more complicated Higgs structures [1]), and the use of richer gauge groups, left-right symmetric models $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [2], or additional $U(1)$ groups $SU(2)_L \otimes U(1) \otimes U(1)'$ [3]. However, the origin of the spontaneous symmetry breaking, needed to generate the gauge boson masses, remains unclear.

In the MSM with one Higgs doublet [4] there are three would-be Gold stone bosons 'eaten' by the gauge fields, which become massive in the process and leave one neutral scalar particle: the Higgs boson. The current experimental lower bound for the mass ($m_H$) of this boson stands at about 60 GeV [5]. The actual value of $m_H$ is crucial for the validity of the MSM at high energies, recalling the fact that the unitarity of the theory is not preserved at high energies if $m_H$ exceeds a critical value of about 1 TeV [6]. In this case the scalar sector of the MSM becomes strongly interacting and the perturbative expansion of the $S$ matrix is no longer valid, thus chiral [7] or effective [8] Lagrangian approaches may be an appropriate description for the gauge bosons physics since they only use the symmetry breaking scheme.

This can be seen if we consider the high energy limit (compared with the mass scale of the particles involved). The polarization vector of a vector boson is

$$\epsilon^\mu = (\frac{\mathbf{k}\cdot\epsilon}{M}, \epsilon + \frac{\mathbf{k}(\mathbf{k}\cdot\hat{\epsilon})}{M(Q+M)}) \tag{1}$$

with $K = (Q, \mathbf{k})$ and $K^2 = M^2$, where it is easy to see that, at high energies, the dominant part of the gauge bosons is that of longitudinal polarization which, in the same limit, can be written as

$$\epsilon^\mu \simeq \frac{K^\mu}{M}\,. \tag{2}$$

Moreover, in the 't-Hooft - Feynman gauge (a special case of the so called $R_\xi$ gauges which we will discuss later in a more general form), the gauge fixing

term is given by

$$\partial_\mu V_i^\mu + iM\phi_i = 0, \tag{3}$$

where $V_i$ and $\phi_i$ are any vector boson field and its corresponding Goldstone boson, one can replace $V_i$ with $\phi_i$ in any $S$ matrix calculation at high energies,

$$S[V_i] = S[\phi_i] + S[\mathcal{O}(\mathcal{M}/\mathcal{Q})] \tag{4}$$

therefore the scattering amplitudes for longitudinal $W$'s and $Z$'s can be calculated from the scattering amplitudes of the would-be Goldstone bosons [9], up to order $M/Q$ (only in this gauge); this is known as the Equivalence Theorem, proposed by Lee, Quigg and Thacker is 1977.

The main result of this paper is to present an ansatz for obtaining the corresponding would-be Goldstone bosons in the $SU(2)_L \otimes U(1)_Y \otimes U(1)'$ model, with one $SU(2)$ doublet and two singlets. We also find the upper bound for the mass of the Higgs bosons in the limit $\sqrt{s} \gg m_H$ by looking at the scattering process of two neutral Goldstone bosons. Via the equivalence theorem this is a good approximation for the scattering amplitude of two neutral high energy gauge bosons, with longitudinal polarization. Finally, we find a relation between the mass of the extra Higgs boson, the new neutral gauge field and the mixing angle between the neutral currents.

The $SU(2)\otimes U(1)\otimes U(1)'$ model is important because future experiments may find additional neutral currents and, even if it is not the exact gauge group of the electroweak interaction, it provides the minimal model with such an extra current, therefore it might be useful when considering some superstring theory-based models, specially those with $E_6$ symmetry or models with left-right symmetry mentioned above. At low energies these models might behave like $SU(2)_L \otimes U(1) \otimes U(1)'$ [15].

The most general expression for the electric charge in $SU(2)_L \otimes U(1) \otimes U(1)'$ is

$$Q = T_{3L} + (aY_1 + bY_2)/2, \tag{5}$$

where $T_{3L}$, $Y_1$ and $Y_2$ are the diagonal generators of $SU(2)_L$, $U(1)$ and $U(1)'$ respectively. The second term is $aY_1 + bY_2 = Y_{GWS}$ for a given multiplet, where $Y_{GWS}$ is the hypercharge in the Weinberg-Salam model. The most general Lagrangian for the bosonic sector in this model with one $SU(2)$ doublet and one singlet is given by

$$L \;=\; \sum_{i=1,2} \Big( \, (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) + \lambda_i \, [\, \Phi_i^\dagger \, \Phi_i - v_i^2 \,]^2 \, \Big)$$

$$+ \quad \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2), \tag{6}$$

where the Higgs field components are:

$$\begin{aligned}
\Phi_1 &= \begin{pmatrix} (\varphi_1 + i\varphi_2)/\sqrt{2} \\ (H + i\varphi_3)/\sqrt{2} \end{pmatrix} \ , \\
\Phi_2 &= (\chi + i\varphi_4)/\sqrt{2} \ ,
\end{aligned} \tag{7}$$

and the covariant derivative is [11]

$$D_\mu = \partial_\mu - ig\vec{T}.\vec{A}_\mu - i\frac{g_1}{2}Y_1 B_\mu - i\frac{g_2}{2}Y_2 C_\mu. \tag{8}$$

The Higgs fields have vacuum expectation values

$$\begin{aligned}
< H >_o &= v_1 \ , \\
< \chi >_o &= v_2,
\end{aligned} \tag{9}$$

and then the symmetry is spontaneously broken to $U(1)_Q$ .

The mass matrix obtained from the Higgs potential for the fields $\varphi_3$, $\varphi_4$ (the neutral would-be Golstone bosons) is identically zero, and it is unknown which of the fields are 'eaten' by the longitudinally polarized gauge fields. To get the renormalizable theory, in the $R_\xi$, gauge we need to cancel the mixing terms $\eta_Z\,(\partial_\mu Z^\mu)$, $\eta_{Z'}(\partial_\mu Z^{\mu'})$ and $\varphi^+\,(\partial_\mu W^{\mu-})$ from the kinetic lagrangian of the Higgs fields, where $\eta_Z$, $\eta_{Z'}$ and $\varphi^+$ are the Goldstone bosons of the $Z_\mu$, $Z'_\mu$ and $W^+_\mu$, respectively. The gauge fixing term in the lagrangian is [12]

$$\begin{aligned}
L_{GF} = \quad &- \quad \frac{1}{2\xi_a}\left(\partial_\mu A^\mu\right)^2 - \frac{1}{2\xi_z}\left(\partial_\mu Z^\mu - \xi_z M_Z \eta_Z\right)^2 \\
&- \quad \frac{1}{2\xi_{z'}}\left(\partial_\mu Z^{\mu'} - \xi_{z'} M_{Z'}\eta_{Z'}\right)^2 - \frac{1}{\xi_w}\left|\partial_\mu W^{\mu+} - i\xi_w \, M_W \, \varphi^+\right|^2 \ ,
\end{aligned} \tag{10}$$

and, using the covariant derivative as a function of the real fields [11], we get

$$\begin{aligned}
\eta_Z &= \frac{g}{M_Z}\frac{\sin\psi}{\cos\theta}\sum_{i=3,4} v_i\left(t_{3i} - \frac{\sin\theta\cot\psi}{\sin 2\xi}[aY_{1\varphi_i} - 2Y_{GWS_i}\sin^2\xi]\right)\varphi_i \ , \\
\eta_{Z'} &= \frac{g}{M_{Z'}}\frac{\cos\psi}{\cos\theta}\sum_{i=3,4} v_i\left(t_{3i} + \frac{\sin\theta\tan\psi}{\sin 2\xi}[aY_{1\varphi_i} - 2Y_{GWS_i}\sin^2\xi]\right)\varphi_i
\end{aligned} \tag{11}$$

where $M_Z$ and $M_{Z'}$ are the $Z_\mu$ and $Z'_\mu$ masses respectively and $t_{3i}$ are the $T_{3L}$ quantum numbers of the Higgs fields. The angles $\theta, \psi$ and $\xi$ are given by

$$\begin{aligned}
\tan\xi &= \frac{a}{b}\frac{g_2}{g_1} , \\
\sin\theta &= \frac{e}{g} , \\
\tan 2\psi &= -\frac{4M_W^2/\cos^2\theta}{M_{Z'}^2 + M_Z^2 - 2M_W^2/\cos^2\theta}\,\frac{\sin\theta}{\sin^2\xi}\,(aY_1 - \sin^2\xi) \qquad (12)
\end{aligned}$$

where $g1$, $g2$ and $e$ are the coupling constants associated with $U(1)$, $U(1)'$ and $U(1)_Q$ respectively, $\theta$ is the Weinberg angle and the angle $\psi$ gives the mixing between the weak neutral currents. If $\psi = 0$ there is no mixing and we get the SM with one additional multiplet plus one extra term due $Z'$. According to some recent results from LEP and the L3 collaboration data [13] the possible values of the mixing angle are in the range $|\psi| \leq 0.03 - 0.01$.

From Eq. (7) and the quantum numbers of the Higgs multiplets,

$$\begin{aligned}
\varphi_3 &= \alpha\,\eta_Z + \beta\,\eta_{Z'}, \\
\varphi_4 &= \rho\,\eta_Z + \sigma\eta_{Z'} \qquad (13)
\end{aligned}$$

in the limit when the mixing angle between $Z_\mu$ and $Z'_\mu$ is $\psi \approx 0$, we have:

$$\begin{aligned}
\alpha &= -\frac{2\cos\theta\cos\psi M_Z}{gv_1} \approx -1 , \\
\rho &= \frac{2\cot\theta M_Z}{gaY_{1\phi_4}v_2}\left[-\sin\xi\cos\xi\sin\psi + \sin\theta\cos\psi(aY_{1\phi_3} - 2\sin^2\xi)\right] \\
&\approx \frac{M_Z\sin\psi}{M_{Z'}} , \\
\beta &= -\frac{2\cos\theta\sin\psi M_{Z'}}{gv_1} - \approx \frac{M_{Z'}\sin\psi}{M_Z} , \\
\sigma &= \frac{2\cot\theta M_{Z'}}{gaY_{1\phi_4}v_2}\left[\sin\xi\cos\xi\cos\psi + \sin\theta\sin\psi(aY_{1\phi_3} - 2\sin^2\xi)\right] \\
&\approx \cos\psi \qquad (14)
\end{aligned}$$

Now let us consider the processes $\eta_Z\eta_Z \to \eta_Z\eta_Z$ and $\eta_{Z'}\eta_{Z'} \to \eta_{Z'}\eta_{Z'}$, the scattering amplitudes are:

$$T(\eta_Z\eta_Z \to \eta_Z\eta_Z) = - \frac{i\alpha^4 m_H^2}{v_1^2}\left[\frac{s}{s-m_H^2} + \frac{t}{t-m_H^2} + \frac{u}{u-m_H^2}\right]$$

$$
\begin{aligned}
& - \frac{i\rho^4 m_\chi^2}{v_2^2}\left[\frac{s}{s-m_\chi^2}+\frac{t}{t-m_\chi^2}+\frac{u}{u-m_\chi^2}\right] , \\
T(\eta_{Z'}\eta_{Z'}\to\eta_{Z'}\eta_{Z'}) = & - \frac{i\sigma^4 m_\chi^2}{v_2^2}\left[\frac{s}{s-m_\chi^2}+\frac{t}{t-m_\chi^2}+\frac{u}{u-m_\chi^2}\right] \\
& - \frac{i\beta^4 m_H^2}{v_1^2}\left[\frac{s}{s-m_H^2}+\frac{t}{t-m_H^2}+\frac{u}{u-m_H^2}\right] , \qquad (15)
\end{aligned}
$$

where $m_H, m_\chi$ are the neutral Higgs bosons masses. Recalling that the mixing angle between the neutral gauge bosons is small and for simplicity we assume that the mixing between $H$ and $\xi$ is zero, i.e. $\lambda_3 = 0$.

The scattering amplitude can be decomposed in partial waves, according to

$$
T(s,\theta) = 16\pi \sum_{j=0}^{\infty} a_j\, P_j(\cos\theta) , \qquad (16)
$$

at high energy ($s >> m_H^2, m_\chi^2$) the tree-level contributions to the $j = 0$ partial wave amplitudes are given by

$$
\begin{aligned}
a_0(\eta_Z\eta_Z\to\eta_Z\eta_Z) &= -\frac{3i}{8\sqrt{2}\pi}\left[\frac{\alpha^4 m_H^2}{v_1^2}+\frac{\rho^4 m_\chi^2}{v_2^2}\right] , \\
a_0(\eta_{Z'}\eta_{Z'}\to\eta_{Z'}\eta_{Z'}) &= -\frac{3i}{8\sqrt{2}\pi}\left[\frac{\sigma^4 m_\chi^2}{v_2^2}+\frac{\beta^4 m_H^2}{v_1^2}\right] . \qquad (17)
\end{aligned}
$$

From unitarity requirement the upper bound for the mass of the Higgs is therefore,

$$
\begin{aligned}
m_H^2 &\leq \frac{4\pi v_1^2}{3\alpha^4} \approx \frac{4\pi\sqrt{2}}{3G_F\cos^4\psi} , \\
m_\chi^2 &\leq \frac{4\pi v_2^2}{3\sigma^4} \approx \frac{4\pi v_2^2}{3\cos^4\psi} \\
&\leq \frac{4\pi \sin^2 2\xi M_{Z'}^2}{3\sqrt{2}G_F \sin^2\theta M_Z^2 \cos^4\psi} , \qquad (18)
\end{aligned}
$$

where we have made use of the mass relation for $M_{Z'}$ obtained in Ref.[11], given by

$$
M_{Z'}^2 = \frac{g^2 v_2^2 \tan^2\theta}{4\sin^2 2\xi} . \qquad (19)
$$

An interesting case occurs when one considers the limit $m_H \leq m_\chi$,

$$m_H^2 \leq m_\chi^2 \leq \frac{4\pi \sin^2 2\xi M_{Z'}^2}{3\sqrt{2} G_F \sin^2\theta M_Z^2} \ . \tag{20}$$

where we find a relation between the $m_\chi$ and the $\xi$ angle for different values of $M_{Z'}$.

In Fig. 1, the region between the horizontal line and the 'parabolic'curves give the possible values for the $m_\chi$ and the $\xi$ angle. For example, if $\xi = \pi/4$ then the $m_\chi = 1728$ GeV, 3458 GeV and, 6916 GeV for $M_{Z'} = 150$ GeV, 300 GeV and 600 GeV respectively.

In conclusion, a relation between the masses and the mixing angles of the gauge fields and the would-be Goldstone bosons, for a theory with one additional neutral current, is found, using the $R_\xi$ gauge fixing, and we are able to recover the MSM constraints on the Higgs mass in the $\psi \approx 0$ case. The latter relation is valid for any extra neutral current and the differences depend on the coupling constants $g1$, $g2$ and the coefficients $a$, $b$ which define the electromagnetic charge. This coefficients depend on the fermionic content of the model and the cancellation of the anomalies.

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# References

[1] J.F. Gunion, H.E. Haber: Nucl. Phys. **B** 272 (1986) 1; J.A. Grifolds, A. Méndez: Phys. Rev. **D 22** (1985) 1725; J.F. Gunion, H.E. Haber, G. Kane, S. Dawson: The Higgs Hunter's Guide (Addison-Wesley Publising Company, Reading, MA, 1990); J.F. Donoghue, Ling Fong Li: Phys. Rev. **D 19** (1979) 945.

[2] C.S. Lim, R. Inami: Prog. Theor. Phys. **67** (1982) 1569; J.F. Gunion, J. Grifolds, A. Méndez, B. Kayser, F. Olness: Phys. Rev. **D 40** (1989) 1546.

[3] K. Kang, J.E. Kim: Phys. Rev. **D 14** (1976) 1903; H. Fritzch, P. Minkowski: Nucl. Phys. **B 103** (1976) 61; N.G. Deshpande, P. Iskandar: Phys. Rev. Lett. **42** (1979) 20; M. Yasue: Prog. Theor. Phys. **61** (1979) 269.

[4] S. Glashow: Nucl. Phys. **22** (1961) 579; S. Weinberg: Phys. Rev. Lett. **19** (1967) 1264; A. Salam: in *Elementary Particle theory* (nobel Symposium No. 8), edited by N. Svartholm (Almquist and Wiksell, Stockholm, 1968).

[5] J.L. Díaz-Cruz and A. Méndez. Nucl. Phys. **B 380** (1992)39.

[6] B.W. Lee, C. Quigg, H.B. Thacker: Phys. Rev. Lett. **38** (1977) 883; M. Veltman: Phys. Lett. **B 70** (1977) 253; M. Veltman: Acta Phys. Polon. **B 8** (1977) 475.

[7] S. Weinberg: Physica **96 A** (1979) 327; J. Gasser and H. Leutwyler: Ann. Phys. (N.Y.) 158 (1984) **142**; A. Dobado, M.J. Herrero: Phys. Lett. **B 228** (1989) 495; J.F. Donoghue, C. Ramirez, G. Valencia: Phys. Rev. **D 38** (1988) 2195.

[8] S. Coleman, J. Wess, B. Zumino: Phys. Rev. **177** (1968) 2239; C. Callan, S. Coleman, J. Wess, B. Zumino: *ibid.*, **177** (1968) 2247.

[9] M. Chanowitz, M. Golden, H. Georgi: Phys. Rev. Lett. **57** (1986) 2344; M. Chanowitz, M. Golden, H. Georgi: Phys. Rev. **D 36** (1986) 1490.

[10] J.M. Cornwall, D.N. Levin, G. Tiktopoulos: Phys. Rev. **D 10** (1974) 1145; B.W. Lee, C. Quigg, H.B. Thacker: Phys. Rev. **D 16** (1977) 1519; L.E. Vayonakis: Lett. Nuovo Cim. **17** (1976) 383; M. Chanowitz, M. Gaillard: Nucl. Phys. **B 261** (1985) 379.

[11] W. Ponce: Phys. Rev. **D 36** (1987)962.

[12] G. 't Hooft: Nucl. Phys. **B 33** (1971) 173; *ibid.* **33** (1971) 167; K. Fujikawa, B.W. Lee, A.I. Sanda: Phys. Rev. **D 6** (1972) 2923.

[13] A. Chiappinelli: Phys. Lett. **B 263** (1991) 287; G. Altarelli *et al., ibid.* **263** (1991) 459; K. T. Mahanthappa, P.K. Mohapatra: Phys. Rev. **D 43** (1991) 3093; P. Langaker: Phys. Lett. **B 256** (1991) 277.

[14] H. Arthur Weldon: Phys. Rev. **D 30** (1984) 1547.

[15] F. del Aguila, M. Cvetič and P. Langacker: Phys. Rev. **D 48** (1993) 969. L3 Collaboration: Cern preprint CERN PPE/93-44. A. Leike, S. Riemann and T. Riemann: preprint CERN-TH 6545/92.

FIG. 1 The region between the horinzontal line and the 'parabolic' curves gives the allowed values for the $m_\chi$ and the mixing angle $\xi$, for different values of $M_{Z'}$. The dashed, the doted and, the dot - dashed curves correspond to the $M_{Z'} = 150$ GeV, 300 GeV and 600 GeV respectively.